CONJUGATE NONSTATIONARY HEAT TRANSFER FOR A THIN PLATE IN THE FLOW OF AN INCOMPRESSIBLE LIQUID

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We obtain the solution in the whole plane of the flow for a nonstationary conjugate problem in heat transfer for a plate in an incompressible liquid.

Let us consider the aerodynamic heating of a semi-infinite plate. At the external surface of the plate $y = 0$ the parallel incompressible flow of a liquid is incident. The thermal flux across the inner surface of the plate $y = -h$ is specified (Fig. 1). For $t = 0$ there are sources inside the plate which are functions of time and the coordinates. This does not affect the stationarity of the hydrodynamic flow since the flow is incompressible. The temperature field of the plate and liquid is nonstationary.

The nonstationary heat transfer problem was considered as a conjugate problem in Lykov's sense by Perel'man [1-2]. To obtain the solution of problem in the whole field of the flow, apart from the leading edge of the plate $(x = y = 0)$, the complete system of Navier–Stokes equations was used in addition to the complete heat conduction equation for an incompressible liquid and the heat conduction equation for the body:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\cdot\frac{\partial p}{\partial x}+\nu\Delta u, \tag{1}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\cdot\frac{\partial p}{\partial y}+\nu\Delta v, \tag{2}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{3}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\lambda}{c\rho}\Delta T+\frac{\mu}{c\rho}\Phi, \tag{4}$$

where the dissipative function Φ is given by

$$\Phi=2\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2\right]+\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^2,$$
$$\rho_p c_p\frac{\partial T_p}{\partial t}=\lambda_p\left(\frac{\partial^2 T_p}{\partial x^2}+\frac{\partial^2 T_p}{\partial y^2}\right)+Q(x, y, t) \tag{5}$$

with the initial and boundary conditions:

$$u=v=0 \text{ for } y=0,\ x>0, \tag{6}$$

$$u\to U_\infty,\ v\to 0 \text{ for } y\to\infty, \tag{7}$$

$$v=0,\ \frac{\partial u}{\partial y}=0 \text{ for } y=0,\ x<0, \tag{8}$$

$$\lambda\left(\frac{\partial T}{\partial y}\right)\Bigg|_{\substack{y=0\\x>0}}=\lambda_p\left(\frac{\partial T_p}{\partial y}\right)\Bigg|_{\substack{y=0\\x>0}}, \tag{9}$$

$$\lambda\left(\frac{\partial T}{\partial y}\right)\Bigg|_{\substack{y=0\\x<0}}=0, \tag{10}$$

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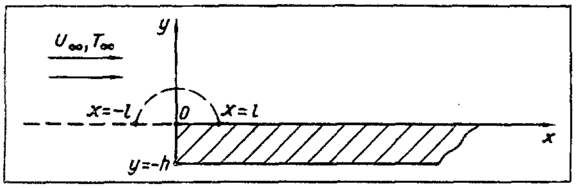

Fig. 1. Physical model and coordinate system.

$$\lambda_p\left(\frac{\partial T_p}{\partial y}\right)\Bigg|_{\substack{y=-h\\x>0}} = F(x,\ t). \tag{11}$$

The function F(x, t) is specified and independent of the time for $t < 0$,

$$T = T_p \quad \text{for } y=0,\ x>0, \tag{12}$$

$$T(x,\ y,\ t) \to T_\infty \quad \text{for } y \to \infty, \tag{13}$$

$$\lambda_p\left(\frac{\partial T_p}{\partial x}\right)\Bigg|_{x=0} = 0; \tag{14}$$

$T(x,\ y,\ 0) = Z(x,\ y)$ is the temperature field in the whole flow before the inclusion of sources ($Z(x,\ y)$ is found below). We seek the solution of the problem (1)-(14) in the domain $x^2 + y^2 \geq l^2$, where l is a dimension of the order of the mean free path of a molecule since Eqs. (1)-(14) are meaningful in domains with typical dimensions much greater than l. The problem (1)-(3) was solved in [3-5] in the above domain with the boundary conditions (6)-(8) in the boundary layer approximation using parabolic coordinates $x = \xi^2 - \eta^2$, $y = 2\xi\eta$, since, by using these coordinates in the boundary layer approximation, we can take account of the effect of the boundary layer on the basic flow and avoid having to join up the solutions in different parts of the flow.

We solve the thermal problem in the liquid by using parabolic coordinates and the solution of the hydrodynamic problem obtained in [3-5]. In (4) we put $u = \partial\psi/\partial y$, $v = -\partial\psi/\partial x$ and transform to nondimensional variables:

$$x,\ y,\ t \to \xi,\ \eta,\ t \to \bar{\xi} = \frac{1}{\sqrt{L}}\ \xi,\ \bar{\eta} = \left(\frac{2U_\infty}{\nu}\right)^{1/2} \eta,\ \bar{\tau} = \frac{2U_\infty}{L}\ t$$

letting $\mathrm{Re} \to \infty$ to obtain the usual boundary layer approximation. We note that in the explicit expression for $\Phi(\xi,\ \bar{\eta})$ terms with negative powers of ξ may be dropped for the hydrodynamic problem was solved in [3-5] in this approximation. After this simplification, the Eq. (4) and its boundary conditions take the following forms in the variables ξ, $\bar{\eta}$, t:

$$\frac{2\xi^2}{U_\infty}\cdot\frac{\partial T}{\partial t} + \xi f'\ \frac{\partial T}{\partial \xi} - f\ \frac{\partial T}{\partial \bar{\eta}} = \frac{1}{\mathrm{Pr}}\cdot\frac{\partial^2 T}{\partial \bar{\eta}^2} + \frac{U_\infty^2}{c}\ (f'')^2, \tag{15}$$

where $f(\bar{\eta})$ is the solution of the problem

$$\frac{d^3 f}{d\bar{\eta}^3} + f\ \frac{d^2 f}{d\bar{\eta}^2} = 0;\ f(0) = f'(0) = 0;\ f'(\infty) = 1;$$

$$f(\bar{\eta}) = \frac{a_1\bar{\eta}^2}{2!} - \frac{a_1^2\bar{\eta}^5}{5!} + \ldots \quad \text{for small } \bar{\eta};\ a_1 = 0.4696\ldots;$$

$$f(\bar{\eta}) = \bar{\eta} - \beta + O\left[\frac{1}{\bar{\eta}^2}\ \exp\left[-\frac{1}{2}\ (\bar{\eta} - \beta)^2\right]\right] \quad \text{for large } \bar{\eta};$$

$$\beta = 1.2168\ \ldots;$$

$$\frac{\lambda}{2\xi}\left(\frac{2U_\infty}{\nu}\right)^{1/2}\left[\frac{\partial T}{\partial \bar{\eta}}\right]_{\bar{\eta}=0} = \lambda_p\left[\frac{\partial T_p}{\partial y}\right]_{\substack{y=0\\x>0}}, \tag{16}$$

$$\frac{\lambda}{2\bar{\eta}}\left(\frac{2U_\infty}{\nu}\right)^{1/2}\left[\frac{\partial T}{\partial \xi}\right]_{\xi=0} = 0; \tag{17}$$

$$\lambda_p\left(\frac{\partial T_p}{\partial x}\right)_{x=0} = 0; \tag{17'}$$

$$T|_{\bar{\eta}=0} = T_p|_{y=0}, \tag{18}$$

$$T(\xi,\ \bar{\eta},\ t) \to T_\infty \quad \text{as } \bar{\eta} \to \infty, \tag{19}$$

$$T(\xi,\ \bar{\eta},\ 0) = Z(\xi,\ \bar{\eta}),$$

$$\lambda_p\left(\frac{\partial T_p}{\partial y}\right)\Bigg|_{\substack{y=-h\\x>0}} = F(x,\ t). \tag{20}$$

We average Eq. (15) over the thickness of the plate:

$$\rho_p c_p \frac{\partial T_{av}}{\partial t} = \lambda_p \frac{\partial^2 T_{av}}{\partial x^2} + \frac{\lambda_p}{h}\left(\frac{\partial T_p}{\partial y}\right)\Bigg|_{\substack{y=0\\x>0}} - \frac{\lambda_p}{h}\left(\frac{\partial T_p}{\partial y}\right)\Bigg|_{\substack{y=-h\\x>0}} + Q_{av}(x, t), \tag{21}$$

where $T_{av} = 1/h \int_{-h}^{0} T_p(x, y, t)dy$, $Q_{av} = 1/h \int_{-h}^{0} Q(x, y, t)dy$. We replace T_{av} by $T_{p|y=0}$, since the plate is assumed to be thin. As a result of this approximation and the use of the boundary conditions (16), (18), and (20), Eq. (21) is transformed to the form

$$\frac{\partial T|_{\bar{\eta}=0}}{\partial t} = a\,\frac{\partial^2 T|_{\bar{\eta}=0}}{\partial x^2} + R(x, t) + A\,\frac{1}{\xi}\left(\frac{\partial T}{\partial \bar{\eta}}\right)_{\bar{\eta}=0}, \tag{22}$$

where

$$a = \frac{\lambda_{av}}{c_p \rho_p};\quad A = \frac{\lambda}{2h\rho_p c_p}\left(\frac{2U_\infty}{\nu}\right)^{1/2};$$

$$R(x, t) = \frac{1}{\rho_p c_p}\left[Q_{av}(x, t) - \frac{1}{h}F(x, t)\right].$$

Let R(x, t) be represented as a generalized power series:

$$R(x, t) = \sum_{n=0}^{\infty} R_n^*(t)\,x^{\frac{n}{k}} = \sum_{m=0}^{\infty} R_{0m}^*(t)\,x^m + \cdots + \sum_{m=0}^{\infty} R_{k-1\,m}^*(t)\,x^{\frac{k-1}{k}+m}.$$

Since Eqs. (22), (15) are linear, it is sufficient to solve them for

$$R(x, t) = \sum_{k=0}^{\infty} R_k(t)\,x^{\rho+k},\quad 0 \leqslant \rho < 1,$$

where $R_k(t)$ = const for $t < 0$. We seek the solution of the problem in the form

$$T(\xi, \bar{\eta}, t) = \Theta(\xi, \bar{\eta}, t)\exp\left\{-\int_0^{\bar{\eta}} \frac{\Pr}{2} f d\bar{\eta}\right\} + T_2(\bar{\eta}), \tag{23}$$

where $T_2(\bar{\eta})$ satisfies the equation

$$\frac{d^2 T_2}{d\bar{\eta}^2} + \Pr f\,\frac{dT_2}{d\bar{\eta}} = -\Pr\,\frac{U_\infty^2}{c}\,(f'')^2$$

with the boundary conditions

$$\frac{dT_2}{d\bar{\eta}}\Bigg|_{\bar{\eta}=0} = 0,\quad T_2(\bar{\eta}) \to T_\infty \ \text{ as } \ \bar{\eta} \to \infty.$$

$T_2(\bar{\eta})$ is the solution of a problem similar to Polhausen's problem, but in the variable $\bar{\eta}$. $T_2(\bar{\eta})$ gives the temperature distribution in the whole flow (apart from the region $x^2 + y^2 = (\xi^2 + (\nu/2U_\infty)\bar{\eta}^2)^2 \geq l^2$), while the Polhausen problem defined the temperature field only above the plate. We note that $T_1 = T_2(0)$ satisfies (5) for $Q(x, y, t) \equiv 0$ and $F(x, t) \equiv 0$. Consequently, in this case $Z(\xi, \bar{\eta}) \equiv T_2(\bar{\eta})$ (from the definition of $Z(\xi, \bar{\eta})$). The function $\Theta(\xi, \bar{\eta}, t)$ is defined by the solution of the following problem:

$$\frac{\partial^2 \Theta}{\partial \bar{\eta}^2} - \left(\frac{\Pr f'}{2} + \frac{\Pr^2 f^2}{4}\right)\Theta = \Pr\left[\frac{2\xi^2}{U_\infty}\cdot\frac{\partial \Theta}{\partial t} + \xi f'\,\frac{\partial \Theta}{\partial \xi}\right], \tag{24}$$

$$\frac{\partial \Theta|_{\bar{\eta}=0}}{\partial t} = a\,\frac{\partial^2 \Theta|_{\bar{\eta}=0}}{\partial x^2} + R(x, t) + A\,\frac{1}{\xi}\left(\frac{\partial \Theta}{\partial \bar{\eta}}\right)_{\bar{\eta}=0}, \tag{25}$$

$$\left(\frac{\partial \Theta}{\partial \xi}\right)_{\xi=0} = 0, \tag{26}$$

$$\left(\frac{\partial \Theta|_{\bar{\eta}=0}}{\partial x}\right)_{x=0} = 0, \tag{26'}$$

$$\Theta \exp\left\{-\int_0^{\bar\eta} \frac{\Pr f}{2}\, d\bar\eta\right\} \to 0 \quad \text{as} \quad \bar\eta \to \infty. \tag{27}$$

We seek the solution of the problem (24)-(27) in the form of a series:

$$\Theta = \sum_k Y_k(\bar\eta,\ t)\, \xi^{2(\rho+k+1)} + \sum_l \Phi_l(\bar\eta,\ t)\, \xi^{2(\rho+l)+1}. \tag{28}$$

(The limits of summation are defined below.)

Substituting (28) in (24), we find that

$$\frac{\partial^2 \varkappa_k}{\partial \bar\eta^2} - \left(\frac{\Pr f'}{2} + \frac{\Pr^2 f^2}{4}\right)\varkappa_k = \Pr\left(\lambda_0 \frac{\partial \varkappa_{k-1}}{\partial t} + \mu_k f' \varkappa_k\right). \tag{29}$$

The equations for $Y_k(\bar\eta,\ t)$ and $\Phi_l(\bar\eta,\ t)$ are combined in (29). They are obtained by the change of variables

$$\lambda_0 \to \frac{2}{U_\infty}, \quad \begin{array}{l} \varkappa_k \to Y_k, \quad \mu_k \to 2(\rho + k + 1), \\ \varkappa_k \to \Phi_k, \quad \mu_k \to 2\left(\rho + k + \frac{1}{2}\right). \end{array} \tag{30}$$

Noting that $x = \xi^2$ for $\bar\eta = 0$, we substitute (28) in (25). Equating the coefficients of like powers of x we have

$$\frac{dy_k}{dt} = a(\rho+k+3)(\rho+k+2)\, y_{k+2} + R_{k+1} + A\bar\varphi_{k+1} \quad \text{for} \quad x^{\rho+k+1}, \tag{31}$$

$$\frac{d\varphi_k}{dt} = a(z_0+k+2)(z_0+k+1)\,\varphi_{k+2} + A\bar y_k \quad \text{for} \quad x^{\rho+k+\frac{1}{2}}, \tag{32}$$

where

$$\begin{gathered} y_k(t) = Y_k|_{\bar\eta=0};\ \bar y_k(t) = \left(\frac{\partial Y_k}{\partial \bar\eta}\right)_{\bar\eta=0}; \\ \varphi_k(t) = \Phi_k|_{\bar\eta=0};\ \bar\varphi_k(t) = \left(\frac{\partial \Phi_k}{\partial \bar\eta}\right)_{\bar\eta=0}. \\ z_0 = \rho + 1/2. \end{gathered} \tag{33}$$

To obtain the equations linking Y_k and $\bar y_k$, φ_k and $\bar\varphi_k$, we have to use (29). Equation (29) has two solutions. After determining an asymptotic expression for $\varkappa_k$ as $\bar\eta \to \infty$, and estimates of the heat flux $Q = \lambda\int_0^{+\infty}(\partial T/\partial y)_{x=-c^2}dy$, we can show that of the two solutions of (29) only that for which $\varkappa_k \sim c_k(t)/\sqrt{\bar\eta}\exp\{-\int_0^{\bar\eta}(\Pr f/2)d\bar\eta\}$ as $\bar\eta \to \infty$ is suitable. It defines the temperature field $T \sim \hat u(\xi,\ t)/\sqrt{\bar\eta}\exp\{-\int_0^{\bar\eta}\Pr f d\bar\eta\}$ which is physically meaningful as $\bar\eta \to \infty$. Let us find the solution as $\bar\eta \to 0$. We note that $f^2 \sim O(\bar\eta^4)$, $f' \sim a_1\eta + O(\bar\eta^3)$ as $\bar\eta \to 0$. Then, to any accuracy of $\bar\eta^3$, we can rewrite (29) as

$$\frac{\partial^2 \varkappa_k}{\partial \bar\eta^2} - \Pr a_1\left(\mu_k + \frac{1}{2}\right)\bar\eta\varkappa_k = \lambda_0 \Pr \frac{\partial \varkappa_{k-1}}{\partial t}. \tag{34}$$

Equation (34) has two solutions, one of which $\to 0$, while the other $\to \infty$ as $\bar\eta \to \infty$. In Eqs. (29) and (34) the coefficients of $\varkappa_k$ are monotonic functions and since

$$\Pr\left[\left(\mu_k + \frac{1}{2}\right)f' + \frac{\Pr f^2}{4}\right] > \Pr\left(\mu_k + \frac{1}{2}\right)a_1\bar\eta,$$

the increasing solution of (29) increases more rapidly than the increasing solution of (34), while the decreasing solution of (34) decreases more slowly than the decreasing solution of (29). Hence the relation between $\varkappa_k(0)$ and $(\partial\varkappa_k/\partial\bar{\eta})_{\bar{\eta}=0}$ for the decreasing solution of (34) is little different from the corresponding relation between $\varkappa_k(0)$ and $(\partial\varkappa_k/\partial\bar{\eta})_{\bar{\eta}=0}$ for the decreasing solution of (29).

If we make the change of variable $z = [a_1 \Pr(\mu_k + 1/2)]^{1/3}\bar{\eta}$ in (34), and solve the resulting equation by the method of varying the arbitrary constants, we find the solution which tends to zero as $z \to \infty$:

$$\varkappa_k = \frac{\Pr \lambda_0}{W_0\left[a_1 \Pr\left(\mu_k + \frac{1}{2}\right)\right]^{2/3}}\left\{\left[\int_0^z \frac{\partial \varkappa_{k-1}}{\partial t} Ai(\zeta)\, d\zeta\right] Bi(z) - \left[\int_0^z \frac{\partial \varkappa_{k-1}}{\partial t} Bi(\zeta)\, d\zeta\right] Ai(z)\right\} + C_k(t)\, Ai(z), \tag{35}$$

where

$$Ai(z),\ Bi(z) \text{ are Airy functions; } W_0 = \begin{vmatrix} Ai(z) & Bi(z) \\ A'i(z) & B'i(z) \end{vmatrix} = \text{const.}$$

From (35), noting (30) and (33), we obtain

$$\begin{aligned} \frac{\bar{y}_k(t)}{y_k(t)} &= \frac{\left\{a_1 \Pr\left[2(\rho+k) + \frac{5}{2}\right]\right\}^{1/3} A'i(0)}{Ai(0)} \equiv p_k, \\ \frac{\bar{\varphi}_k(t)}{\varphi_k(t)} &= \frac{\left\{a_1 \Pr\left[2(\rho+k) + \frac{3}{2}\right]\right\}^{1/3} A'i(0)}{Ai(0)} \equiv q_k. \end{aligned} \tag{36}$$

The form of the recurrence relations (31)-(32) for $k < 0$, noting that $R_k = 0$ (for $k < 0$), noting the conditions (26), (26'), and that $1/\xi(\partial\Theta/\partial\bar{\eta})_{\bar{\eta}=0}$ is bounded as $x \to 0$, shows that $\varphi_l = 0$, $l < 3$; $y_k = 0$, $k < 1$. Writing out the recurrence relations (31) for $k = 0, 2, \ldots, 2(m-1)$, noting (36), we apply the operator $a^{i-1}\Gamma(\rho + 2i)\, d^{m-i}/dt^{m-i}$ to the i-th row and add all the relations. Then

$$y_{2m} = -\sum_{i=1}^{m} \frac{a^{i-1}\Gamma(\rho + 2i)}{a^m \Gamma(\rho + 2m + 2)} \cdot \frac{d^{m-i}}{dt^{m-i}} (A q_{2i-1}\varphi_{2i-1} + R_{2i-1}). \tag{37}$$

Similarly, for odd k, we find that

$$y_{2m+1} = -\sum_{i=0}^{m} \frac{a^{i-1}\Gamma(\rho + 2i + 1)}{a^m \Gamma(\rho + 2m + 3)} \cdot \frac{d^{m-i}}{dt^{m-i}} (A q_{2i}\varphi_{2i} + R_{2i}). \tag{38}$$

Substituting (37) and (38) in (32) yields the following for even k:

$$\varphi_{2(m+1)} = \left[\frac{d\varphi_{2m}}{dt} + A p_{2m} \sum_{i=1}^{m} \frac{a^{i-1}\Gamma(\rho + 2i)}{a^m \Gamma(\rho + 2m + 2)} \frac{d^{m-i}}{dt^{m-i}} (A q_{2i-1}\varphi_{2i-1} + R_{2i-1})\right] \Big/ \left[a(z_0 + 2m + 2)(z_0 + 2m + 1)\right] \tag{39}$$

and the following for odd k:

$$\varphi_{2m+3} = \left[\frac{d\varphi_{2m+1}}{dt} + A p_{2m+1} \sum_{i=0}^{m} \frac{a^{i-1}\Gamma(\rho + 2i + 1)}{a^m \Gamma(\rho + 2m + 3)} \frac{d^{m-i}}{dt^{m-i}} (A q_{2i}\varphi_{2i} + R_{2i})\right] \Big/ \left[a(z_0 + 2m + 3)(z_0 + 2m + 2)\right]. \tag{40}$$

All the functions $y_k(t)$, $k = 1, 2, \ldots$ and $\varphi_l(t)$, $l = 3, 4, \ldots$ are determined from the recurrence relations (37)-(40) in terms of the $R_k(t)$-coefficients of the source.

If in (35) instead of $C_k(t)$ we substitute $\varphi_k(t)/Ai(0)$ and $y_k(t)Ai(0)$, we find $\Phi_k(\bar{\eta}, t)$ and $Y_k(\bar{\eta}, t)$ respectively for small $\bar{\eta}$. Using (28) and (23), we determine the temperature field $T(\xi, \bar{\eta}, t)$ for small $\bar{\eta}$.

We write down from (16), (18), (23), (28), (33), and (36) the expressions for the temperature at the surface of the plate and the thermal flux across it:

$$T_p|_{y=0} = \sum_{k=1}^{\infty} y_k(t)\, x^{\rho+k+1} + \sum_{l=3}^{\infty} \varphi_l(t)\, x^{\rho+l+\frac{1}{2}} + T_l,$$

$$q = \frac{\lambda}{2}\left(\frac{2U_\infty}{\nu}\right)^{1/2}\left[\sum_{k=1}^{\infty} p_k y_k(t)\, x^{\rho+k+\frac{1}{2}} + \sum_{l=3}^{\infty} q_l \varphi_l(t)\, x^{\rho+l}\right],$$

where $T_l = T_2(0)$. We note that in the case of sources which are independent of the time, $\varphi_k(t)$ = const, $y_k(t)$ = const, $R_k(t)$ = const, and the recurrence relations (31)-(32) take the form

$$y_{k+2} = -\frac{Aq_{k+1}\varphi_{k+1} + R_{k+1}}{a(\rho+k+3)(\rho+k+2)},$$

$$\varphi_{k+2} = -\frac{Ap_k y_k}{a(z_0+k+2)(z_0+k+1)}.$$

This solution of the stationary problem defines the field $Z(\xi, \overline{\eta})$.

NOTATION

u	is the liquid velocity in the x direction;
v	is the liquid velocity in the y direction;
U_∞	is the velocity of the incident flow;
T	is the liquid temperature;
T_p	is the plate temperature;
T_∞	is the temperature of the incident flow;
$L = \mathrm{Re}\nu/U_\infty$	is an appropriate parameter in the x direction;
h	is the plate thickness;
λ_p	is the heat transfer coefficient of the plate;
λ	is the heat transfer coefficient of the liquid;
a	is the thermal diffusivity coefficient of the plate;
μ	is the dynamic viscosity coefficient for the liquid;
ν	is the kinematic viscosity coefficient for the liquid;
c_p	is the heat capacity of the plate;
c	is the heat capacity of the liquid;
ρ_p	is the plate density;
ρ	is the liquid density;
Pr	is the Prandtl number;
Re	is the Reynolds number;
$Q(x, y, t)$	is the specific intensity of the sources;
$\psi(x, y)$	is the stream function;
p	is the liquid pressure.

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